AP Calculus AB

Course Description:

This is a comprehensive year-long course in the study of both differential and integral calculus and is intended to be the equivalent of a college level Calculus I course. Students will be studying the ideas of functions, graphs, limits, derivatives, integrals and the Fundamental Theorems of Calculus as outlined in the AP Calculus Course description (as it appears on the AP Central website). The intent is for students to master the fundamentals of calculus in order to succeed on the AP Calculus AB exam and be adequately prepared to be successful in higher mathematics courses.

Students should have mastery of material including the study of algebra, geometry, coordinate geometry, trigonometry, analytic geometry, pre-calculus, and elementary functions (linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions). Students must also be familiar with the properties, algebra, graphs, and language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on). Additionally, students should know the values of the trigonometric functions at the standard intervals (and their multiples).

Students should already have the ability to use a graphing utility to perform basic operations (graph a function in an appropriate viewing window, approximate zeroes, approximate extrema, find points of intersection, identify asymptotes, etc). Material will be presented using the TI-83 Edition calculator and students are encouraged to use this model, or another equivalent model approved for use on the AP exam. If a student cannot afford a graphing calculator, they will be able to check one out from the media center. Calculators must be brought to class every day as its use is an integral part of the course. By the end of the year, students should be able to use a calculator to graph a function in any window, determine the value of a derivative at a specified point, find the value of a definite integral, solve an equation, and intelligently analyze and interpret results. Calculators are NOT permitted on all assessments.

Course Outline:

Functions, Limits, Continuity (4 weeks, 1 test)

- Review of pre-calculus functions, their graphs, domain
- Graphical introduction to limits
- Limit theorem, one sided limits, limits at infinity
- Continuity of a function at a number, on an interval
- Continuity of a composite function
- Continuity of trigonometric functions
- Intermediate Value Theorem

Teaching Notes:
During this first unit of calculus, I spend extra time making sure each student is familiar with their graphing calculator. Most students use the Tl - 83 Plus but several will also have the Tl- 89's. In all topics I try to emphasize that concepts must be understood analytically, numerically and graphically. With limits I place less time now teaching the definition and formal proof of a limit and more time discussing the "getting close to" concept through the graphing calculator and tables of values.

Sample Activity:
In the Point of Discontinuity Activity, the students will explore limits at discontinuities in four different ways: using the table feature on their calculator, using algebraic techniques to simplify the expressions, using the trace feature on their calculators, and using verbal descriptions of functions written in words to create graphs that match the verbal descriptions. In this activity the students must compare and contrast the different types of discontinuities in a written summary and will also give an oral presentation describing how each method reveals discontinuities.

Sample Activity:
In the "Take It to the Limit" Essay, students must find an example of a song, poem, picture, or other "real-life" item and explain how it uses or illustrates the concept of a limit.

Communication:
Part of each class period students are allowed to work together on calculus problems. At this time, students are encouraged to explain and justify their understanding of the calculus to other students and to me. Some problems worked on the board for all to see and questions the answers are probed for depth of understanding.

The Derivative (5 1/2 weeks, 2 tests)

- Slope of a tangent line
- Definition of the derivative
- Differentiation and continuity
- Numerical derivative
- Basic differentiation rules for polynomials
- Higher order derivatives
- The derivative as a rate of change
- Derivatives of trig functions, composite functions, chain rule derivatives of power functions
- Implicit differentiation

Teaching Notes:

By introducing the concept of the derivative as the slope of the tangent line first, I find the students get a nice geometric (visual) handle on this most important concept. We use the graphing calculators to see what the derivative looks like and especially what it looks like when a function is not differentiable.

Sample Activity:

In the Derivative at a Point Activity the students will use a worksheet to explore a derivative function using the limit definition of the derivative at a point. For several points, students compute the difference quotient algebraically using the definition and use a table and graph on their calculator to evaluate the limits and interpret their results in terms of the definition to decide if the derivative exists or not at each point. The original function and its derivative values are plotted on two graphs with the same horizontal axes. Students will trade papers and validate each other's results.

Sample Activity:

The Students will complete a derivative worksheet that gives a table of values for f, f', g, and g'. This worksheet asks students to evaluate expressions such as the derivative of $f(g(x))$ at $x=3$.

Sample Activity:

In the "How Many Licks?" Lab the students determine the rate of change of the volume of a spherical lollipop as it is consumed. Using calipers, students measure the initial diameter of the lollipop, and then measure every 30 seconds as they suck on the lollipop. Using their generated data and a graphing calculator, they must find a curve of best fit. This equation is then used in various calculations concerning related rates. The students will turn in a formal Lab Report.
Related Rates Project

Connecting concepts & Processes:

Throughout the course we will continually revisit ideas, concepts, and problems using a variety of representations (graphical, numerical, written, and verbal explanations) to tie together the idea that all concepts can be explained using a variety of different methods. I begin early in the year having my students justify their answers on quizzes and tests. I prefer that they write at least one sentence explaining their reasoning rather than using a chart or number line. I do stress that being able to verbally communicate using math terminology from day one of class is an important goal of this course. Just doing the math is never enough. Demonstrating and explaining the math is also important.

Applications of Differentiation (5 weeks, 2 tests)

- Related rates
- Particle motion
- Maximum and Minimum function values
- Rolles' Theorem and Mean Value Theorem
- Increasing, Decreasing functions, First Derivative Test
- Concavity, Points of Inflection, Second Derivative Test
- Limits at infinity, describing asymptotic behavior using limits
- Summary of Curve Sketching, relating f, f' and f"
- Optimization Problems, both relative and absolute extrema
- Newton's method, Linear approximations,
- Differentials

Teaching Notes:

Particle motion problems are introduced here and offer a good time to review parametric equations. Parametric equations are graphed on the calculator to show the motion of the particle. Graphing calculators are de-emphasized somewhat in this unit to show students how they can find the graph of a function without it. By this time in the year students become over- dependent on their calculators and now must learn to do without them at times.

In most application problems, the author's problems are in verbal form. Students are expected to translate these into differential equations, solve and then return their answers into a verbal analysis of the original problem.

Sample Activity:

Students will be given a worksheet on the Mean Value Theorem. This worksheet contains a set of functions on specified domains and the students must determine whether they can apply the Mean Value Theorem. The students will get together in pairs to compare their answers before having a class discussion.

Sample Activity:

The Calculus Coaster is a project that combines the concepts of continuity and differentiation and applies it to a real world situation. The students must first use both continuity and differentiation to solve for multiple coefficients. They then use the coefficients they have solved for and their knowledge of derivatives to answer questions about the situation

Sample Activity:

In the Searching for $f(x)$ Activity, students will use the graph of the derivative $f'(x)$ to

determine the key features of *f(x)*. These features include increasing/decreasing intervals, local extrema, points of inflection, and concavity. They will use these key features to create a graph of *f(x)*. In the second part of this activity, the students will exchange their graphs of *f(x)* with a different group and make a graph of *f'(x)* to match their new *f(x)*. The groups will then meet together to check *f'(x)* graphs and resolve any differences.

The Definite Integral, Integration, & the Fundamental Theorems of Calculus

(5 1/2 weeks, 2 tests)

- Antidifferentiation
- Riemann sums and Definite Integrals
- First Fundamental Theorem of Calculus part 1
- Second Fundamental Theorem of Calculus part 2
- Area by rectangular approximations and trapezoidal approximations
- Average value of a function
- Numeric integration on the calculator

Teaching Notes:

Techniques of integration I try to de-emphasized and put more emphasis on Riemann sums and area under curve as a visual reminder of what integration is doing. Just as the slope of the tangent line gives the student a geometric handle on the derivative, area under the curve helps students see integration as an adding up of areas of rectangles --- a sum in general.

Sample Activity:

In the Fundamental Theorem of Calculus Lab, students are given a lab of past-free response questions in which they must use the Fundmentals Theorem of Calculus. Within these problems, they are often required to calculate a definite integral with their calculators. In addition, they must answer the questions about extrema and inflection points if *g* using calculus if given a function $g(x) = \int_a^x f(t)dt$ and a graph of *f*.

Sample Activity:

In the Computing Areas Using Calculus Methods Activity the students will complete a series of worksheets students to verify their skills on area computations.

Students will:

a) trace given shapes in XY coordinates

b) determine the function equations

c) determine the limits of integration

d) use TI-84 to find the areas
e) verify their computations with geometry formulas.

Logarithmic, Exponential & Other Transcendental Functions (5 weeks , 2 tests)

- Defining the Natural Logarithmic Function
- Logarithmic differentiation and integration
- Defining the Natural Exponential Function
- Derivatives and Antiderivatives of the Natural Exponential Functions
- Other Exponential and Logarithmic Functions, their derivatives and antiderivative
- Inverse Trig functions: differentiation and integration

Teaching Notes:
After the natural log function and exponential functions are defined, this unit is a good review of the differentiation and integration from earlier units. The chain rule, quotient rule, applications of the derivative and integration that have been taught earlier are again seen but with these new functions as their basis.

Differential Equations and Applications of Integration (4 weeks, 1 test)

- Differential Equations - first and second order
- Slope fields, Euler's method
- Area using the definite integral
- Volume by disk, shell, slicing, and washer methods
- Growth and Decay applications
- More on limits and L'Hospital's Rule

Sample Activity:
Students will be given a worksheet on L'Hospital's Rule. On this worksheet the students must determine if L'Hospital's Rule applies to a given limit and then apply the rule to evaluate the limit analytically, if appropriate.

Sample Activity:
In the Growth and Decay Activity students are given a variety of growth and decay word problems where the rate of change of the dependent variable is proportional to the same variable. Students are asked to translate the problem situation into a differential equation using proper notation. Students show the steps in solving the differential equation, continuing to use proper notation for each step. In the second part of this activity the students will vary initial conditions and use their calculators to graph the resulting solutions so that students can explore the effect of these changes.

Sample Activity:

In the Volume Activity students will work in pairs to find the volume of 3-D honeycomb paper party decorations. Students will trace the decoration, measure the outline to construct a data table, and use regression to determine the curves of best fit for their object. They will write the integrals to represent the volume of their objects as solids of revolution and compute the volumes on their calculators. Pairs will assess the reasonableness of answers and compare results with another team.

AP REVIEW:

I try to leave 2 or 3 weeks before the AB exam for review time. During this time the students work independently on multiple-choice and free response questions from AP released exams. Students are expected to talk to each other about the questions and solutions but must write their own solutions and justifications for me.

Textbook: Larson, Hostetler, Edwards. *Calculus of a Single Variable.* 8th ed. Boston: Houghton Mifflin, 2006

Student Evaluation:

Quarter grades are computed using homework, quizzes and tests. Each quarter grade represents 40% of the semester grade. The final exam represents 20% of the grade. An assignment sheet is given with each unit outlining the homework problems that are to be done for each section. * These problems are discussed each day with emphasis on both written and verbal justification of their work. I find this is the best time to check for in-depth understanding or misunderstanding of my teaching. A student at this level should be doing homework because they understand its importance not because it may be collected. Therefore, I collect only about one-half of the assignments for a grade. Quizzes and Tests will be announced and will always and will cover 3 or 4 sections of the unit. I rarely give a test over an entire unit because to the time limitations and volume of material to be tested. Calculators will be allowed on some parts of the tests and not on other parts. As early as possible I start incorporating AP type questions (both multiple-choice and free response) into the unit test. The semester exams and final AP review rely heavily on previous AP exams.

Homework:

Homework is assigned daily. Solutions to homework problems are found graphically, numerically, analytically, and verbally in order to demonstrate knowledge of the calculus curriculum being studied. Proper symbolism and vocabulary is expected to be used in the classroom and on all assignments. On all work, solution alone will not be given credit. Answers must be accompanied by the appropriate work.

Quizzes & Tests:
Each assessment will contain a variety of question types: multiple choice, short answer, free response. Thus, you will have to express your answers in a number of different ways. Some responses may require a sketch along with a written explanation and others may require a verbal interpretation of the set-up of the problem. Tests will include any material that the instructor has taught from chapter 1 including any material from previous courses. They will also include Free-Response questions from previous AP Calculus Exams. Calculators are NOT permitted on all parts of assessments.

Teacher Resources:

Primary Textbook: Larson, Hostetler, Edwards. *Calculus of a Single Variable .* 8th ed. Boston: Houghton Mifflin, 2006

Supplemental Material:
Leithold. *The Calculus 7.* 7th ed. New York, HarperColins Publishers. Inc.

Lin McMullin. *Multiple Choice & Free Response Questions in Preparation for the AP Calculus (BC) Examination.* 6th ed. D & S Marketing Systems